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# Analyzing the influence of treatment awareness rate on COVID-19 pandemic by fractional derivative-based modeling and simulation

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Covid-19 disease is a respiratory illness caused by SARS-Cov-2 and poses a serious public health risk. It usually spread from person-to-person. The fractional- order of covid-19 was determined and basic reproduction number using the next generation matrix was calculated. The stability of disease-free equilibrium and endemic equilibrium of the model were investigated. Also, sensitivity analysis of the reproduction number with respect to the model parameters were carried out. It was observed that in the absence of infected persons, disease free equilibrium is achievable and is asymptotically stable. Numerical simulations were presented graphically. The results of the model analysis indicated that  $R_0 < 1$  is adequate enough to reducing the spread of disease and disease persevere in the population when  $R_0 > 1$  The numerical results showed that effective vaccination of the population helps in curtailing the spread of the viral disease.

In order to know whether the disease may die out or persist, basic reproduction number,  $R_0$  was obtained using Next Generation Matrix Method. It was observed that the value of  $R_0$  is high when the depletion of awareness programme is high while the value of  $R_0$  is very low when the rate of implementation of awareness programme is high. So, neglecting the implementation of awareness program can have serious effect on the population. The model shows the implementation of awareness program is the key eradication to the pandemic.

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**Keywords:** COVID-19; Public enlightenment; Laplace Adomian decomposition method; Fractional derivative; Numerical simulation.

## 1. Introduction

A global outbreak of COVID-19 has taken place, with new outbreaks emerging in Wuhan, China in December 2019. When a large number of patients were admitted to hospitals in late December 2019 with pneumonia, it was clear that the epidemic had started [18]. At first, doctors in Wuhan, Hubei Province, China suspected the seafood and wet animal market to be the cause. According to the World Health Organization (WHO), COVID, also known as Coronavirus Disease 2019, is caused by the Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) virus [1]. In March 2020, the World Health Organization (WHO) declared the COVID-19 pandemic. The index case was confirmed in Nigeria on February 27, 2020, leading to the establishment of a multi-sectoral Emergency Operations Center by the Nigerian Center for Disease Control (EOC), with a total of 11,516 cases and 323 deaths reported [1].

Professionals have proposed various hypotheses to explain the peculiar behaviors of COVID-19. A mathematical model was found to include undiagnosed infectious cases, hospital sensitization conditions, and a proportion of known cases [16]. A study on the analysis of Mathematical model of COVID-19 incorporated with vaccination and media induced fear was studied in [7]. The

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studies shows that vaccination of susceptible individuals is an effective strategy to curb the spread of COVID-19. The impact of mask use on the general public was investigated using a mathematical model [11]. A new mathematical model of COVID-19, accounting for the effects of first and second doses of vaccination, was presented [5]. The basic reproduction number, an epidemic indicator, was obtained and stability analysis criteria determined. The study concluded that double-dose vaccination was the best way to curb the spread of COVID-19. Researchers also studied the impact of COVID-19 infections on social interactions, showing that social awareness and speedy testing had an impact on a COVID-19 transmission model [21, 17]. The research considered both known and unknown COVID-19 infections during the exposed phase of infection. The influence of public awareness efforts on the dynamics of COVID-19 infection was studied [19]. Advancing in research, [8] analysed the impact of media Campaign on the transmission of COVID-19 epidemics using stochastic model analysis. The effectiveness of the introduced strategy in eradicating the spread of COVID-19 was observed as infected individuals in their population model drastically get reduced. Also, [14] carried out a dynamical analysis and proposed control strategies for capturing the spread of COVID-19 and a conceptual analysis of the concurrent impact of vaccination, treatment and curfew were studied by [15]. Numerical techniques, such as the Adomian Decomposition Method [2], the Variational Iteration Method [13], and the Homotopy Perturbation Method [12] and its modifications[3], are also used by researchers to obtain approximate solutions. The homotopy perturbation method was used to analyze the Equine Infectious Anemia Virus (EIAV) model [6] and to obtain an approximate solution to a fractional-order integral-differential equation [4]. These results converge faster to the exact solution compared to other numerical methods. The future condition of COVID-19 in Bangladesh was discussed by analyzing current data with an SIR model [9]. Also the researches involving the applications of Laplace Adomian Decomposition Method for numerical simulations include [23] who solved an epidemiological mathematical model of COVID-19 using the Laplace Adomian decomposition method to investigate the effect of Caputo-Fractional order in the dynamics of population distribution to the disease prevalence. Research into an effective COVID-19 vaccination continues, with a new mathematical model of the COVID-19 pandemic, including the vaccination campaign, being studied [22]. Vaccine hesitancy, defined as a delay or refusal in accepting vaccination despite its availability, was studied through a cross-sectional survey of health students to determine the determinants of COVID-19 vaccine acceptance and hesitancy [24]. As a motivation and subject of active research, we endeavors to examine the transmission dynamics of COVID-19 in a theoretical population through the development of a mathematical model. Additionally, we aim to evaluate the impact of government-led awareness campaigns on the spread of the virus

## 2. Material and Methods

### 2.1. Model Formulation

We propose a deterministic mathematics model on the transmission dynamics of COVID-19, the population under consideration is divided seven classes, based on the epidemiological status of individual in population, the compartments are Susceptible  $S(t)$ ; These are individuals who are susceptible or prone to COVID-19, Exposed class  $E(t)$ ; These are individuals who are infected with Covid-19 without any signs and symptoms; individuals in this class can infect those in the susceptible class, Infected class  $P(t)$ ; These are individuals who are infected with Covid-19 and are with signs and symptoms, those in this group are capable of transmitting the disease to the susceptible individuals, Quarantine class  $Q(t)$ ; These are individuals who are exposed to Covid-19 and are restricted to prevent the spread of the disease.  $A(t)$  represent individuals who are asymptomatic,  $H(t)$  represent hospitalized individual. The total population size  $N(t)$  is assumed to be constant and well mixed, that is  $N(t) = S(t) + E(t) + A(t) + P(t) + H(t) + Q(t)$ . The model equations are given as follows:

$$\begin{aligned}
 \frac{dS}{dt} &= \Lambda(1 - V) - BSP - \mu S \\
 \frac{dE}{dt} &= BSP - (\mu + \rho + r + \theta)E \\
 \frac{dA}{dt} &= \rho E - (k_1 + \mu)A \\
 \frac{dP}{dt} &= rE - q(1 + y)P + nQ - (\mu + b)P \\
 \frac{dQ}{dt} &= \theta E - (\mu + n + \eta)Q \\
 \frac{dH}{dt} &= q(1 + y)P - (\mu + K_2)H \\
 \frac{dR}{dt} &= \Lambda v + k_1 A + bP + \eta Q + k_2 H - \mu R
 \end{aligned}
 \tag{1}$$

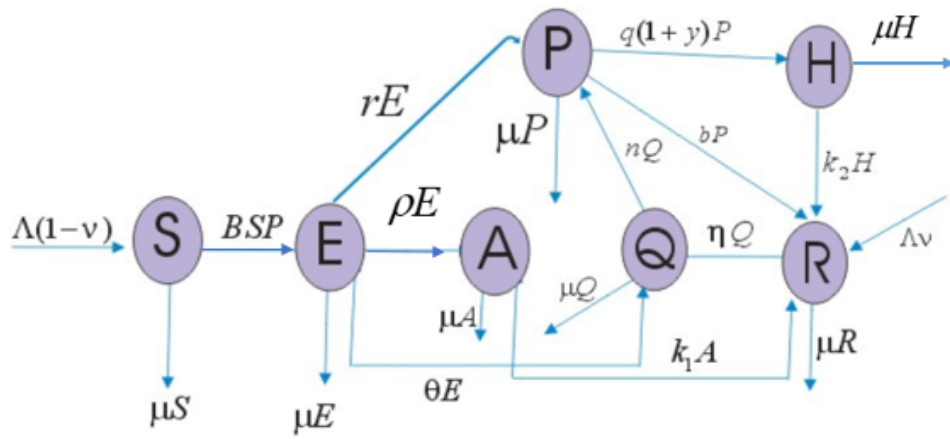


Figure 1: Schematic Diagram of the Proposed COVID-19 Model

Table 1: Variables, parameters with their descriptions

| Variables | Descriptions   |
|-----------|--|
| $S(t)$    | Susceptible individuals.   |
| $E(t)$    | Exposed individuals.   |
| $A(t)$    | Asymptomatic.  |
| $P(t)$    | Infected individuals.  |
| $H(t)$    | Hospitalized   |
| $Q(t)$    | Quarantine individuals   |
| $R(t)$    | Recovered individuals.   |
| $V$       | Rate of Vaccination.   |
| $y$       | Rate of Enlightenment.   |
| $R$       | Rate at which exposed individuals moves the systematic infectious class. |
| $B$       | Effective contact rate of the infection                                  |
| $\gamma$  | Recovery rate  |
| $M$       | Death rate   |
| $\Lambda$ | Recruitment rate   |
| $Q$       | Rate of transmission from P to H dose of vaccine rate                    |
| $N$       | Rate of transmission from P to Q dose of vaccine rate                    |

### 3. Mathematical Analysis.

#### 3.1. Positivity of Solution

**Theorem 1** Given  $S_0 > 0, E > 0, A > 0, P > 0, Q > 0, H > 0$  and  $R > 0$ . Then the solution  $\{ (S, E, A, P, H, Q, R \in \mathbb{R}^7 : N \leq \frac{\Lambda}{\mu}) \}$  are positive invariant for  $t \geq 0$

**proof.** Recall from (1),

$$\frac{dS}{dt} = \Lambda(1 - V) - BSP - \mu S$$

therefore,

$$\begin{aligned} \frac{dS}{dt} &\geq -(\mu + Bp)S \\ \frac{dS(t)}{S(t)} &\geq -(\mu + BP)dt \end{aligned} \tag{2}$$

Solving (2) via separation of variable technique with the application of the initial condition, yields;

$$S(t) \geq S_0 e^{-(\mu+BP)t} \geq 0 \tag{3}$$

Repeating the same procedure for the rest of the equations we have,

$$E(t) \geq E_0 e^{-(\rho+\theta+r+\mu)t} \geq 0 \tag{4}$$

$$A(t) \geq A_0 e^{-(k_1 + \mu + \delta_1)t} \geq 0 \tag{5}$$

$$E(t) \geq E_0 e^{-(\mu + \lambda + \rho)t} \geq 0 \tag{6}$$

$$P(t) \geq P_0 e^{-(\mu + \delta_2 + b + q + \alpha y)t} \geq 0 \tag{7}$$

$$Q(t) \geq Q_0 e^{-(\mu + n + \eta)t} \geq 0 \tag{8}$$

$$H(t) \geq H_0 e^{-(\mu + \delta_3 + K_2)t} \geq 0 \tag{9}$$

$$R(t) \geq R_0 e^{-\mu t} \geq 0 \tag{10}$$

Therefore, the model's solution yields a positive outcome. This concludes the proof of the theorem, as it satisfies all fundamental characteristics of an epidemiology model. Consequently, we can assert that the suggested model is appropriate for investigating COVID-19 within the human population.  $\square$

### 3.2. Existence and Uniqueness of Solution

The Lipchitz criterion will be employed to verify the existence and uniqueness of the solution. Thus from (2),

Let:

$$\begin{aligned} g_1 &= \Lambda(1 - V) - BSP - \mu S \\ g_2 &= (\mu + \rho + r + \theta)E \\ g_3 &= \rho E - (k_1 + \mu)A \\ g_4 &= rE - q(1 + y)P + nQ - (\mu + b)P \\ g_5 &= \theta E - (\mu + n + \eta)Q \\ g_6 &= q(1 + y)P - (\mu + K_2)H \end{aligned} \tag{11}$$

**Theorem 2** Let  $E^1$  represent the region  $0 \leq w \leq R$ , then the system of (1) has a unique solution, provided that  $\frac{\partial f_i}{\partial x_j}, i = 1, 2, \dots, 6$  are bounded and continuous.

**proof.** From (1) we obtain the following partial derivative given below:

$$\begin{aligned} \left| \frac{\partial g_1}{\partial S} \right| &= \left| \frac{-BP - \mu}{1 + \alpha P} \right| < \infty, \left| \frac{\partial g_1}{\partial E} \right| = |0| < \infty, \left| \frac{\partial g_1}{\partial A} \right| = |0| < \infty, \left| \frac{\partial g_1}{\partial P} \right| = \left| \frac{-BS}{(1 + \alpha I)^2} \right| < \infty, \left| \frac{\partial g_1}{\partial Q} \right| = |0| < \infty, \left| \frac{\partial g_1}{\partial H} \right| = |0| < \infty, \\ \left| \frac{\partial g_1}{\partial R} \right| &= |0| < \infty \end{aligned}$$

For  $g_2 = \frac{BSP}{1 + \alpha P} - (\mu + \rho + r + \theta)E$

$$\begin{aligned} \left| \frac{\partial g_2}{\partial S} \right| &= \left| \frac{BP}{1 + \alpha P} \right| < \infty, \left| \frac{\partial g_2}{\partial E} \right| = |-(\mu + \rho + \delta_1)| < \infty, \left| \frac{\partial g_2}{\partial A} \right| = |0| < \infty, \left| \frac{\partial g_2}{\partial P} \right| = \left| \frac{BS}{(1 + \alpha I)^2} \right| < \infty, \left| \frac{\partial g_2}{\partial Q} \right| = |0| < \infty, \\ \left| \frac{\partial g_2}{\partial H} \right| &= |0| < \infty, \left| \frac{\partial g_2}{\partial R} \right| = |0| < \infty, \left| \frac{\partial g_3}{\partial S} \right| = |0| < \infty, \left| \frac{\partial g_3}{\partial E} \right| = |\rho| < \infty, \left| \frac{\partial g_3}{\partial A} \right| = |-(\mu + k_1 + \delta)| < \infty, \left| \frac{\partial g_3}{\partial P} \right| = |0| < \infty, \\ \left| \frac{\partial g_3}{\partial Q} \right| &= |0| < \infty, \left| \frac{\partial g_3}{\partial H} \right| = |0| < \infty, \left| \frac{\partial g_3}{\partial R} \right| = |0| < \infty, \left| \frac{\partial g_4}{\partial S} \right| = |\alpha I| < \infty, \left| \frac{\partial g_4}{\partial E} \right| = |\rho| < \infty, \left| \frac{\partial g_4}{\partial A} \right| = |0| < \infty \\ \left| \frac{\partial g_4}{\partial P} \right| &= |-(\mu + b) - q(1 + y)| < \infty, \left| \frac{\partial g_4}{\partial Q} \right| = |n| < \infty, \left| \frac{\partial g_4}{\partial H} \right| = |0| < \infty, \\ \left| \frac{\partial g_4}{\partial R} \right| &= |0| < \infty, \left| \frac{\partial g_5}{\partial S} \right| = |0| < \infty, \left| \frac{\partial g_5}{\partial E} \right| = |\theta| < \infty, \left| \frac{\partial g_5}{\partial A} \right| = |0| < \infty, \left| \frac{\partial g_5}{\partial P} \right| = |0| < \infty \\ \left| \frac{\partial g_5}{\partial Q} \right| &= |-(\mu + n + \eta)| < \infty, \left| \frac{\partial g_5}{\partial H} \right| = |0| < \infty, \left| \frac{\partial g_5}{\partial R} \right| = |0| < \infty, \left| \frac{\partial g_6}{\partial S} \right| = |0| < \infty, \left| \frac{\partial g_6}{\partial E} \right| = |0| < \infty, \left| \frac{\partial g_6}{\partial A} \right| = |0| < \infty, \\ \left| \frac{\partial g_6}{\partial P} \right| &= |q(1 + y)| < \infty, \left| \frac{\partial g_6}{\partial Q} \right| = |0| < \infty, \\ \left| \frac{\partial g_6}{\partial H} \right| &= |-(\mu + \delta_2 + k_2)| < \infty, \left| \frac{\partial g_6}{\partial R} \right| = |0| < \infty, \left| \frac{\partial g_7}{\partial S} \right| = |0| < \infty, \left| \frac{\partial g_7}{\partial E} \right| = |0| < \infty, \left| \frac{\partial g_7}{\partial A} \right| = |k_1| < \infty \\ \left| \frac{\partial g_7}{\partial P} \right| &= |b| < \infty, \left| \frac{\partial g_7}{\partial Q} \right| = |\eta| < \infty, \left| \frac{\partial g_7}{\partial H} \right| = |k_2| < \infty, \left| \frac{\partial g_7}{\partial R} \right| = |-\mu| < \infty \end{aligned} \tag{12}$$

## 4. Equilibrium Analysis

In this section, the disease free and endemic equilibria of the model will be discussed.

### Disease free equilibrium

At the disease-free equilibrium state of system (1), i.e. when the infection is absent ( $I = 0$ ) and the following result is obtained:

$$(S_0, E_0, A_0, P_0, Q_0, H_0) = \left( -\frac{(-1 + V)\Lambda}{\mu}, 0, 0, 0, 0, 0 \right) \tag{12}$$

**Endemic equilibrium state**

At the endemic equilibrium state, when  $I \neq 0$ , the equilibrium points given by  $E_* = (S^*, E^*, A^*, P^*, Q^*, H^*)$  are obtained as:

$$S^* = \frac{\Lambda\alpha rh + dfh + dhg + \Lambda\alpha n\theta}{(rh + \theta n)(B + \mu\alpha)}$$

$$E^* = \frac{h\Lambda CB r - hd\mu f - hd\mu g + \Lambda CB n\theta}{d(Brh + Bn\theta + \mu\alpha rh + \mu\alpha n\theta)}$$

$$A^* = \frac{\rho(h\Lambda CB r - hd\mu f - hd\mu g + \Lambda CB n\theta)}{d(Brh + Bn\theta + \mu\alpha rh + \mu\alpha n\theta)e}$$

$$P^* = \frac{h(\Lambda CB r - hd\mu f - hd\mu g + \Lambda CB n\theta)}{(f + g)h(B + \mu\alpha)d}$$

$$Q^* = \frac{\theta(h\Lambda CB r - hd\mu f - hd\mu g + \Lambda CB n\theta)}{d(Brh + Bn\theta + \mu\alpha rh + \mu\alpha n\theta)h}$$

$$H^* = \frac{f(h\Lambda CB r - hd\mu f - hd\mu g + \Lambda CB n\theta)}{(f + g)h(B + \mu\alpha)d_j}$$

$$(\mu k_1 \rho j h^2 d j^2 + \mu k_1 \rho j h^2 d f^2 - \mu k_2 \rho e f^2 h^2 d r + k_2 h^2 e f \Lambda C B r^2 + 2 k_2 e f h \Lambda B C r n \theta - k_1 \rho j h^2 \Lambda C B r f$$

$$R^* = \frac{+k_2 n^2 e f \Lambda C B \theta^2}{h e j u (f + g) (B + \mu \alpha) d (r h + n \theta)}$$

Where  $c = 1 - V$ ,  $d = \theta + \rho + \mu + r$ ,  $e = \mu + k_1 + \delta_1$ ,  $f = (1 + y)$ ,  $g = \mu + \delta_2 + b$  and  $h = \mu + n + \eta j = \mu + \delta_3 + k_2$

**5. Basic Reproduction Number**

The basic reproduction number, conventionally denoted by  $R_0$  is defined by Diekmann and Heesterbeek [10] as the average number of secondary infections generated by a typical infectious individual during his or her entire period of infectiousness. Considering three compartments  $E, Q, P$ .

$$E = \frac{BSP}{1 + \alpha P} - (\mu + \rho + r + \theta)E$$

$$P = \rho E - q(1 + y)P + nQ - (\mu + \delta_2 + b)P \tag{13}$$

$$Q = \theta E - (\mu + n + \eta)Q$$

$$F = \begin{bmatrix} 0 & \alpha S & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} (\mu + \rho + r + \theta) & 0 & 0 \\ -\rho & (\mu + \delta_2 + b) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$G = F \times V^{-1} = \begin{bmatrix} \frac{1}{(\mu + \delta_2 + b)(\mu + \rho + r + \theta)} & \frac{-\rho}{(\mu + \delta_2)(\mu + \delta_2 + b)(\mu + \rho + r + \theta)} & 0 \\ 0 & \frac{1}{(\mu + \rho)(\mu + \delta_2 + b)} & \frac{-\theta}{(\mu + \delta)(\mu + \delta_2 + b)(\mu + \rho + r + \theta)} \\ 0 & 0 & \frac{1}{(\mu + \rho)(\mu + \delta + r + \theta)} \end{bmatrix}$$

Computing the spectral radius of matrix  $G$  yields the  $R_0$  given by:

$$R_0 = \frac{\alpha \phi (\omega + \sigma + \mu) \beta}{(\mu + \rho)(\mu + \delta + \omega)(\mu + \delta + \tau + \phi)[\mu(\eta + \omega + \sigma + \mu) + \eta \sigma]} \tag{15}$$

**6. Stability Analysis**

*6.1. Local Stability Analysis of Diseases free Equilibrium*

**Lemma 1:** The disease-free equilibrium of the model is locally asymptotically stable if  $R_0 < 1$  and unstable if  $R_0 > 1$ .

**proof:** We consider the Jacobian of the system of equation (1) which is given by:

$$J(X^0) = \begin{bmatrix} -\mu & 0 & 0 & \frac{-B\Lambda C}{\mu} & 0 & 0 \\ 0 & -d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\rho + \mu) & \alpha S^0 & 0 \\ 0 & 0 & 0 & \rho & -(\phi + \delta + \mu + \tau) & 0 \\ 0 & 0 & 0 & 0 & k_2 & -\mu \\ 0 & 0 & -k_1 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

Then eigenvalues are:

$$\lambda_1 = -\mu - \frac{1}{2}(\rho + \sigma + \tau + \eta) + \frac{1}{2}\sqrt{\rho(\rho - 2\sigma - 2\tau + 4\alpha S^0 - 2\eta) + \eta(\eta + 2\sigma + 2\tau) + \sigma^2 + 2\sigma\tau + \tau^2}$$

$$\lambda_2 = -\mu - \frac{1}{2}(\rho + \delta + \tau + \eta) - \frac{1}{2}\sqrt{\rho(\rho - 2\lambda - 2\tau + 4\alpha S^0 - 2\eta) + \eta(\eta + 2\delta + 2\tau) + \delta^2 + 2\delta\tau + \tau^2}$$

$$\lambda_3 = -\mu - \frac{1}{2}(\omega + \eta - \mu) + \frac{1}{2}\sqrt{\alpha^2 + 2\alpha\eta + 2\alpha\delta + \eta^2 - 2\eta\delta + \delta^2}$$

$$\lambda_4 = -\mu - \frac{1}{2}(\alpha + \eta + \sigma) - \frac{1}{2}\sqrt{\delta^2 + 2\mu\eta + 2\mu\lambda + \eta^2 - 2\eta\sigma + \sigma^2}$$

$$\lambda_5 = -(\mu + \lambda)$$

$$\lambda_6 = -(\mu + k_1 + \mu)$$

Since the eigenvalues are all negatives, we therefore conclude that the disease-free equilibrium state of the Covid-19 model is locally asymptotically stable.

### 6.2. Local Stability Analysis of Endemic Equilibrium

**Theorem:** Endemic equilibrium state is locally asymptotically stable if the determinant of a Jacobian matrix is greater than zero and the trace of the same matrix is less than zero [20]

**proof.** The stability analysis of the Endemic Equilibrium is carried out using the trace and the determinant approach, where the Jacobian  $J^*$  of

$$J^* = \begin{pmatrix} A & \theta & 0 & 0 & G & 0 \\ \eta & B & 0 & 0 & 0 & 0 \\ 0 & \delta & C & 0 & 0 & 0 \\ M & 0 & 0 & D & K & 0 \\ 0 & 0 & 0 & \rho & E & 0 \\ 0 & 0 & 0 & 0 & \varphi & F \end{pmatrix} \tag{17}$$

Where,

$$A = -(\alpha P^* + \eta + \mu), \quad B = -(k_1 + \delta + \mu), \quad C = -(\mu + \lambda), \quad D = -(\rho + \mu), \quad E = -(\theta + \delta + \mu + \tau), \\ F = -(\delta + k + \mu), \quad G = -\alpha S^*, \quad K = \alpha S^*, \quad M = \alpha P^*$$

Hence

$$Det (J^*) = (ABDE - ABK\rho + BGM\rho - DE\eta\alpha + K\eta\delta\rho) CF$$

It is clearly seen that  $\det.(J^*) > 0$ .

Also  $Trace of (J^*) = [-(\alpha P^* + \eta + \mu) - (\rho + \sigma + \mu) - (\mu + \lambda) - (\rho + \mu) - (\theta + \delta + \mu + \tau) - (\delta + k + \mu)]$

That is,  $-[(\alpha P^* + \eta + \mu) + (\omega + \sigma + \mu) + (\mu + \lambda) + (\rho + \mu) + (\phi + \delta + \mu + \tau) + (\delta + k + \mu)] < 0$ .

Hence, the trace of  $(J^*) < 0$ . Thus, the system  $(J^*)$  has eigenvalues that contains negative real parts; therefore, we conclude that the endemic equilibrium system is locally asymptotically stable.  $\square$

### 6.3. Global Stability at Disease Free Equilibrium

To proof the global stability, we make use of Castillo-Chavez method (Castillo et al 2004). Consider a model of the form

$$\begin{cases} \frac{dP}{dt} = D(P, Q) \\ \frac{dQ}{dt} = F(P, Q), F(P, 0) = 0 \end{cases} \tag{18}$$

Where  $P \in R^n$  represents, individuals that are not infected in the population and  $T \in R^n$  represent infected individuals. Following the above representation, the Disease-free equilibrium state can be written as  $T_0 = (P_0, 0)$ , the two conditions given below is used to verify the disease-free equilibrium is globally asymptotically stable.

$$(M_2) F(P, T) = AT - F(P, T), \text{ where } F(P, T) \leq 0 \forall (P, T) \in \Omega$$

Here,  $A = B_v F(P_0, 0)$  denote an M-matrix.

**Lemma 1**  $Q_0 = (P_0, 0)$  is globally asymptotically stable if  $R_0 \leq 1$  and assumption that  $(M_1) - (M_2)$  are satisfied.

**Theorem 3** If  $R_0 < 1$ , the disease-free equilibrium is globally asymptotically stable and unstable if  $R_0 > 1$

**proof.** For  $(M_1)$  we consider  $\frac{dP}{dt} = D(P, 0)$   
 Here

$$D(P, 0) = \begin{bmatrix} \Lambda(1 - V) & -\mu S \\ \frac{BSP}{1+\alpha P} & -(\mu + \rho + r + \theta)E \\ \rho E & -(k_1 + \mu + \delta_1)A \end{bmatrix} \tag{19}$$

The corresponding Jacobian matrix is

$$J_{D(P,0)} = \begin{bmatrix} -(\mu + \eta) & \omega & 0 \\ \eta & -(\omega + \rho + \mu) & 0 \\ 0 & \rho & -(\mu + \Lambda) \end{bmatrix} \tag{20}$$

Characteristics polynomial is given as:

$$\lambda^3 + (\mu + \delta + 2\mu + \Lambda + \eta + \theta)\lambda^2 + (2\mu^2 + 2\delta\eta + 2\mu\eta + 2\theta\mu + \mu\Lambda + \theta\eta + \Lambda\eta + \theta\Lambda + \delta\mu + \delta\Lambda + \mu\Lambda)\lambda\mu^3 + (\alpha + \delta + \Lambda + \eta)\mu^2 + (2\delta\eta + \rho + \Lambda + \rho\Lambda\delta + \sigma\eta + \Lambda\eta)\mu + (2\delta\Lambda + \sigma\Lambda)\eta$$

Next, for  $(M_2)$ , we consider  $\frac{dQ}{dt} = F(P, T)$   
 That is,

$$F(P, T) = \begin{bmatrix} rE & -q(1 + y)P + nQ - (\mu + \delta_2 + b)P \\ \theta E & -(\mu + n + \eta)Q \\ q(1 + y)P & -(\mu + \delta_2 + k_2)H \end{bmatrix} \tag{21}$$

$$F(P, T) = AT - \hat{F}(P, T)$$

Hence, A is M-Matrix and  $\hat{F}(P, T) \geq 0$   
 $\Rightarrow (M_1)$  and  $(M_2)$  Conditions are satisfied. □

#### 6.4. Global Stability of Endemic Equilibrium Point

**Theorem 4** If  $R_0 > 1$ , the endemic equilibrium point of the model equation (1) is globally asymptotically stable

**proof.** To establish the global stability of the endemic equilibrium of the model, the following Lyapunov function can be constructed such that,

$$V(s^*, e^*, a^*, p^*, q^*, h^*) = (S - S^* - S^* \log \frac{S^*}{S}) + (E - E^* - E^* \log \frac{E^*}{E}) + (A - A^* - A^* \log \frac{A^*}{A}) + (P - P^* - P^* \log \frac{P^*}{P}) + (Q - Q^* - Q^* \log \frac{Q^*}{Q}) + (H - H^* - H^* \log \frac{H^*}{H}) \tag{22}$$

The derivative of N along this solution of equation (25) by direct calculation gives:

$$\frac{dN}{dt} = \left(\frac{S - S^*}{S}\right) \frac{dS}{dt} + \left(\frac{E - E^*}{E}\right) \frac{dE^*}{dt} + \left(\frac{A - A^*}{A}\right) \frac{dA^*}{dt} + \left(\frac{P - P^*}{P}\right) \frac{dP}{dt} + \left(\frac{Q - Q^*}{Q}\right) \frac{dQ}{dt} + \left(\frac{H - H^*}{H}\right) \frac{dH}{dt}$$

Where

$$\begin{aligned} \frac{dS^*}{dt} &= \Lambda(1 - V) - \frac{BSP}{1+\alpha P} - \mu S, \\ \frac{dE^*}{dt} &= \frac{BSP}{1+\alpha P} - (\mu + \rho + r + \theta)E, \quad \frac{dA^*}{dt} = \rho E - (k_1 + \mu + \delta_1)A \\ \frac{dP^*}{dt} &= rE - q(1 + y)P + nQ - (\mu + \delta_2 + b)P \\ \frac{dQ^*}{dt} &= \theta E - (\mu + n + \eta)Q, \\ \frac{dH^*}{dt} &= q(1 + y)P - (\mu + \delta_2 + k_2) \end{aligned} \tag{23}$$

Thus, we have

$$\begin{aligned} &\left(\frac{S - S^*}{S}\right) [\Lambda - \Lambda V - \mu S - \frac{BSP}{1+\alpha P}] + \left(\frac{E - E^*}{E}\right) [\frac{BSP}{1+\alpha P} - (\mu + \rho + r + \theta)E] + \left(\frac{A - A^*}{A}\right) [\rho E - (k_1 + \mu + \delta_1)A] + \left(\frac{P - P^*}{P}\right) [rE - q(1 + y)P] \\ &+ \left(\frac{Q - Q^*}{Q}\right) [\theta E - (\mu + n + \eta)Q] + \left(\frac{H - H^*}{H}\right) [q(1 + y)P - (\mu + \delta_2 + k_2)H] \end{aligned} \tag{24}$$

This implies;

$$\begin{aligned}
 & -\frac{(S-S^*)^2}{S} \frac{B(P-P^*)^2}{1+\alpha(P-P^*)^2} - \frac{(S-S^*)^2}{S} \mu - \frac{(E-E^*)^2}{E} (\mu + \rho + r + \theta) - \frac{(A-A^*)^2}{A} (k_1 + \mu + \delta_2) - \frac{(P-P^*)^2}{P} q(1+y)) \\
 & -\frac{(P-P^*)^2}{P} (\mu + \delta_2 + b) - \frac{(Q-Q^*)^2}{Q} (\mu + n + \eta) - \frac{(H-H^*)}{H} (k_2 + \mu + \delta_2) + (\Lambda(1-V)) + \frac{B((S-S^*)(P-P^*))}{1+\alpha(P-P^*)} \\
 & + \rho \frac{(E-E^*)}{E} r(E-E^*) + n(Q-Q^*) + \theta(E-E^*) + q(1+y)(P-P^*)
 \end{aligned} \tag{25}$$

Collecting the positive and negative terms we obtain

$$\frac{dv}{dt} = M - N$$

Where

$$\begin{aligned}
 M = & -\frac{(S-S^*)^2}{S} \frac{B(P-P^*)^2}{1+\alpha(P-P^*)^2} - \frac{(S-S^*)^2}{S} \mu - \frac{(E-E^*)^2}{E} (\mu + \rho + r + \theta) - \frac{(A-A^*)^2}{A} (k_1 + \mu + \delta_2) \\
 & -\frac{(P-P^*)^2}{P} q(1+y)) - \frac{(P-P^*)^2}{P} (\mu + \delta_2 + b) - \frac{(Q-Q^*)^2}{Q} (\mu + n + \eta)
 \end{aligned} \tag{26}$$

Also,

$$\begin{aligned}
 N = & -\frac{(H-H^*)}{H} (k_2 + \mu + \delta_2) + (\Lambda(1-V)) + \frac{B((S-S^*)(P-P^*))}{1+\alpha(P-P^*)} + \rho \frac{(E-E^*)}{E} r(E-E^*) \\
 & + n(Q-Q^*) + \theta(E-E^*) + q(1+y)(P-P^*)
 \end{aligned} \tag{27}$$

If  $M < N$ , Then  $\frac{dv}{dt}$  will be negative.

$\frac{dv}{dt} = 0$ ; if and only if

$$S = S^*, E = E^*, A = A^*, P = P^*, Q = Q^* \text{ and } H = H^*$$

Thus the largest compact invariant set is  $\{(s^*, e^*, a^*, p^*, q^*, h^*) \in \Omega : \frac{dv}{dt} = 0\}$  is just the singleton set  $\{E^*\}$  Thus the endemic equilibrium, by LaSalle's Invariant principles; it implies that  $E^*$  is Globally Asymptotically Stable (GAS) in  $\Omega$  if  $M < N$ .  $\square$

## 7. Numerical solution and Simulations

### Laplace Adomian Decomposition Method (LADM)

In this section, the Laplace-Adomian decomposition method is employed to solve the system of differential equations, and the obtained approximate solution of the model is applied to conduct numerical simulations.

#### Definition 1:

Let  $f(t)$  be a given function defined for all positive real numbers  $t \geq 0$ , the following properties hold for Laplace transform:

- The Laplace transform of  $f(t)$  is the function  $f(s)$  given by:  $f(s) = \int_0^\infty e^{-st} f(t) dt$
- The Laplace transform of function  $f(t)$  with order  $\eta$  is defined as

$$L[f^\eta(t)] = \alpha^\eta L[f(t)] - \alpha^{\eta-1} L[f(0)] - \alpha^{\eta-2} L[f'(0)] - \alpha^{\eta-3} L[f''(0)]$$

- The inverse Laplace transform of function  $\frac{f(s)}{s}$  is given by:

$$L^{-1} \frac{f(s)}{s} = \int_0^t f(t) dt$$

#### Definition 2:

The Adomian polynomials, is a recursive formula given by  $A_n(t) = \sum_{n=0}^\infty \frac{1}{n} (t \frac{d}{dt}) A_{n-1}(t)$ .

#### Model solution via LADM

Consider system (1) given by

$$\begin{aligned}
 \frac{d^{T_1} S(t)}{dt} &= \Lambda(1-v) - BSP - \mu S \\
 \frac{d^{T_2} E(t)}{dt} &= BSP - (\mu + \rho + r + \theta) E \\
 \frac{d^{T_3} A(t)}{dt} &= \rho E - (K_1 + \mu) A
 \end{aligned}$$



$$\frac{d^{\tau_4} P(t)}{dt} = rE - q(1 + y)P + nQ - (\mu + b)P \quad (3.53)$$

$$\frac{d^{\tau_5} Q(t)}{dt} = \theta E - (\mu + n + \eta)Q$$

$$\frac{d^{\tau_6} H(t)}{dt} = q(1 + y)P - (\mu + K_2)H$$

Where  $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$  and  $\tau_6$  are the fractional orders.

Taking the Laplace Transform of the equations, we have

$$\begin{aligned} L(D^{\tau_2} E(t)) &= L(BSP - (\mu + \rho + r + \theta)E) \\ L(D^{\tau_3} A(t)) &= L(\rho E - (K_1 + \mu)A) \\ L(D^{\tau_4} P(t)) &= L(rE - q(1 + y)P + nQ - (\mu + b)P) \quad (3.54) \\ L(D^{\tau_5} Q(t)) &= L(\theta E - (\mu + n + \eta)Q) \\ L(D^{\tau_6} H(t)) &= L(q(1 + y)P - (\mu + K_2)H) \end{aligned}$$

Using the property of Laplace Transform and initial conditions to the above equations.

$$\begin{aligned} L(S(t)) &= \frac{1}{s} S_0 + \frac{1}{s^{\tau_1}} L(\Lambda(1 - \nu) - BSP - \mu S) \\ L(E(t)) &= \frac{1}{s} E_0 + \frac{1}{s^{\tau_2}} L(BSP - (\mu + \rho + r + \theta)E) \\ L(A(t)) &= \frac{1}{s} A_0 + \frac{1}{s^{\tau_3}} L(\rho E - (K_1 + \mu)A) \\ L(P(t)) &= \frac{1}{s} P_0 + \frac{1}{s^{\tau_4}} L(rE - q(1 + y)P + nQ - (\mu + b)P) \\ L(Q(t)) &= \frac{1}{s} Q_0 + \frac{1}{s^{\tau_5}} L(\theta E - (\mu + n + \eta)Q) \\ L(H(t)) &= \frac{1}{s} H_0 + \frac{1}{s^{\tau_6}} L(q(1 + y)P - (\mu + K_2)H) \end{aligned} \quad (28)$$

So, writing the solution of each compartment in form of infinite series result to and the non-linear terms can be decomposed by Adomian polynomials:

$$S(t) = \sum_{i=0}^{\infty} S_i, E(t) = \sum_{i=0}^{\infty} E_i, A(t) = \sum_{i=0}^{\infty} A_i, P(t) = \sum_{i=0}^{\infty} P_i, S(t) = \sum_{i=0}^{\infty} S_i, Q(t) = \sum_{i=0}^{\infty} Q_i, H(t) = \sum_{i=0}^{\infty} H_i$$

and  $S(t)\rho(t) = \sum_{i=0}^{\infty} C_i$  where  $C_i = \frac{1}{\sqrt{n+1}} \frac{d^n}{dt^n} [\sum_{k=0}^n \lambda^k \delta_k \sum_{k=0}^n \lambda^k \rho_k] \lambda = 0$   
 Assume,  $S(0) = m_1, E(0) = m_2, A(0) = m_3, P(0) = m_4, Q(0) = m_5, H(0) = m_6$ .  
 Applying the initial value to equation (29) gives:

$$\begin{aligned} \sum_{i=0}^{\infty} L(S_i(t)) &= \frac{m_1}{s} E_0 + \frac{1}{s^{\tau_1}} L(\Lambda(1 - \nu) - B \sum_{i=0}^{\infty} C_n P - \mu S_n) \\ \sum_{i=0}^{\infty} L(E_i(t)) &= \frac{m_2}{s} + \frac{1}{s^{\tau_2}} L(B \sum_{i=0}^{\infty} C_n - (\mu + \rho + r + \theta) E_n(t)) \\ \sum_{i=0}^{\infty} L(A_i(t)) &= \frac{m_3}{s} + \frac{1}{s^{\tau_3}} L(\rho E_n(t) - (K_1 + \mu) A_n(t)) \\ \sum_{i=0}^{\infty} L(P_i(t)) &= \frac{m_4}{s} + \frac{1}{s^{\tau_4}} L((rE_n(t) - q(1 + y)P_n(t) + nQ_n(t) - (\mu + b)P_n(t))) \\ \sum_{i=0}^{\infty} L(Q_i(t)) &= \frac{m_5}{s} + \frac{1}{s^{\tau_5}} L((\theta E_n(t) - (\mu + n + \eta)Q_n(t))) \\ \sum_{i=0}^{\infty} L(H_i(t)) &= \frac{m_6}{s} + \frac{1}{s^{\tau_6}} L(q(1 + y)P_n(t) - (\mu + K_2)H_n(t)) \end{aligned} \quad (29)$$

Comparing both sides of equation (30) gives

$$S_0(t) = m_1, E_0(t) = m_2, A_0(t) = m_3, P_0(t) = m_4, Q_0(t) = m_5, H_0(t) = m_6$$

$$\begin{aligned} L(S_1(t)) &= \frac{-Bm_1 m_4 - m_1 \mu + \lambda(1 - \nu)}{s^{\tau_1 + 1}} \\ L(E_1(t)) &= \frac{Bm_1 m_4 - m_2(\mu + \rho + r + \theta)}{s^{\tau_2 + 1}} \\ L(A_1(t)) &= \frac{m_2 \rho - m_3(k_1 + \mu)}{s^{\tau_3 + 1}} \\ L(P_1(t)) &= \frac{m_5 n - m_2 r + m_4 q(1 + y) - m_4(b + \mu)}{s^{\tau_4 + 1}} \\ L(Q_1(t)) &= \frac{m_2 \theta - m_5(n + \eta + \mu)}{s^{\tau_5 + 1}} \\ L(H_1(t)) &= \frac{m_4 q(1 - y) - m_6(k_2 + \mu)}{s^{\tau_6 + 1}} \end{aligned} \quad (30)$$

Taking the Inverse Laplace Transform of equation (31) gives:

$$\begin{aligned}
 S_1(t) &= (-Bm_1m_4 - m_1\mu + \lambda(1 - v)) \frac{t^{\tau_1}}{\Gamma(\tau_1+1)} \\
 E_1(t) &= (-Bm_1m_4 - m_2(\mu + \rho + r + \theta)) \frac{t^{\tau_2}}{\Gamma(\tau_2+1)} \\
 A_1(t) &= m_2\rho - m_3(k_1 + \mu) \frac{t^{\tau_3}}{\Gamma(\tau_3+1)} \\
 P_1(t) &= m_5n - m_2r + m_4q(1 + y) - m_4(b + \mu) \frac{t^{\tau_4}}{\Gamma(\tau_4+1)} \\
 Q_1(t) &= m_2\theta - m_5(n + \eta + \mu) \frac{t^{\tau_5}}{\Gamma(\tau_5+1)} \\
 Q_1(t) &= m_4q(1 - y) - m_6(k_2 + \mu) \frac{t^{\tau_6}}{\Gamma(\tau_6+1)}
 \end{aligned}
 \tag{31}$$

Also taking  $l = 2, l = 1$  | equation (32) gives

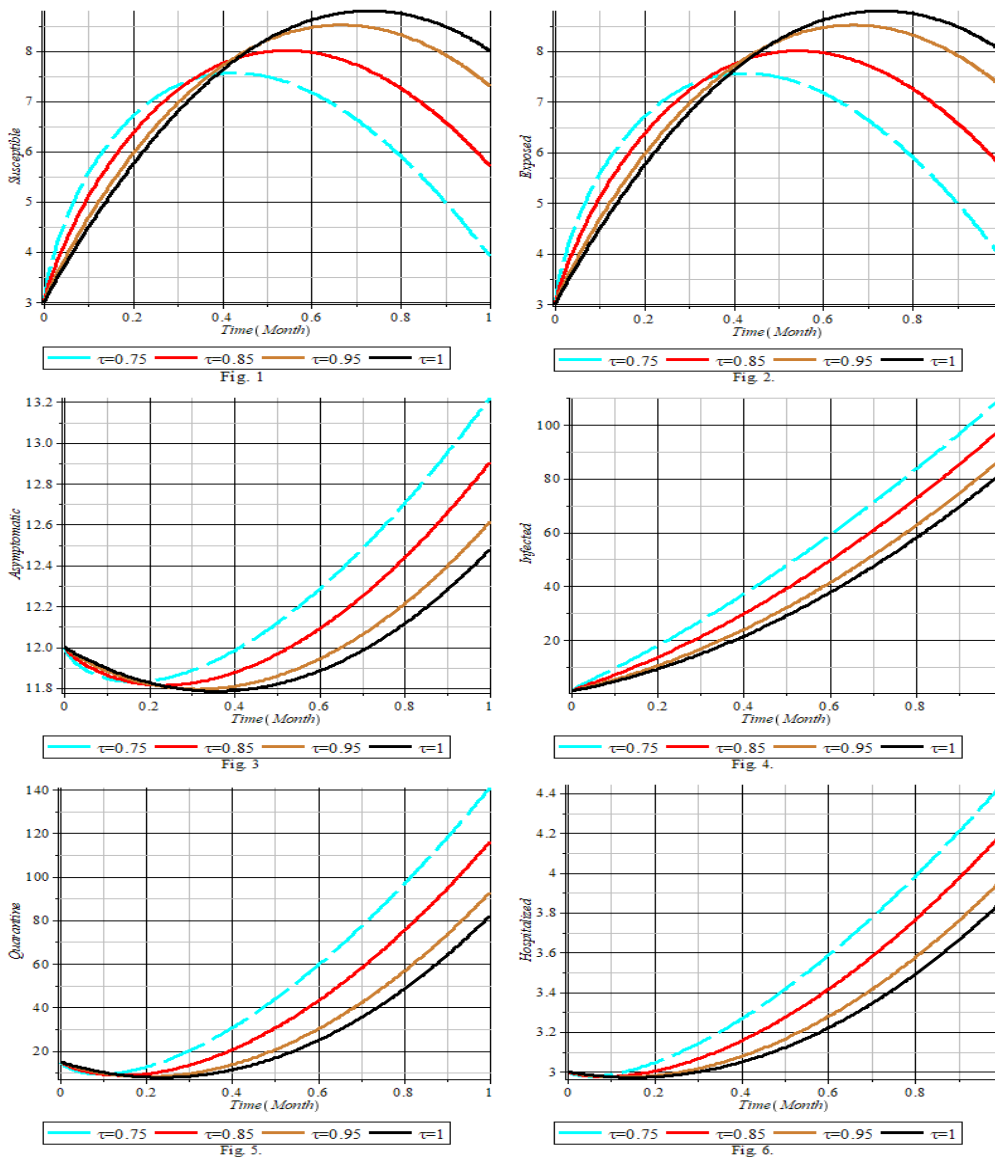
$$\begin{aligned}
 L(S_2(t)) &= \frac{1}{S^{\tau_1}} \left( \mu \frac{(-Bm_1m_4 - m_1\mu + \lambda(1 - v))}{S^{\tau_1+1}} + \frac{1 - v}{S} \lambda \right) \\
 &\quad - \frac{1}{S^{\tau_1}} \left( B \frac{m_5n - m_2r + m_4q(1 + y) - m_4(b + \mu)(-Bm_1m_4 - m_1\mu + \lambda(1 - v))\Gamma(1 + \tau_1 + \tau_4)}{S^{\tau_1 + \tau_4 + 1}\Gamma(1 + \tau_1)\Gamma(1 + \tau_4)} \right) \\
 L(E_2(t)) &= \frac{1}{S^{\tau_2}} \left( \frac{-(\mu + \rho + r + \theta)(Bm_1m_4 - m_2(\mu + \rho + r + \theta))}{S^{\tau_2+1}} \right) + \\
 &\quad \frac{1}{S^{\tau_1}} \left( \frac{Bm_5n - m_2r + m_4q(1 + y) - m_4(b + \mu)(-Bm_1m_4 - m_1\mu + \lambda(1 - v))\Gamma(1 + \tau_1 + \tau_4)}{S^{\tau_1 + \tau_4 + 1}\Gamma(1 + \tau_1)\Gamma(1 + \tau_4)} \right) \\
 L(A_2(t)) &= \frac{1}{S^{\tau_3}} \left( \frac{-(k_1 + \mu)(-m_3(k_1 + \mu) + m_2\rho)}{S^{\tau_3+1}} \right) + \frac{1}{S^{\tau_3}} \left( \frac{\rho(Bm_1m_4 - m_2(\mu + \rho + r + \theta))}{S^{\tau_2+1}} \right)
 \end{aligned}
 \tag{32}$$

$$\begin{aligned}
 L(P_2(t)) &= \frac{1}{S^{\tau_4}} \left( \frac{-(b + q(1 + y) + \mu)m_5n - m_2r + m_4q(1 + y) - m_4(b + \mu)}{S^{\tau_4+1}} \right) + \\
 &\quad \frac{1}{S^{\tau_4}} \left( \frac{n(m_2\theta - m_5(n + \eta + \mu))}{S^{\tau_5+1}} + \frac{r(Bm_1m_4 - m_2(\mu + \rho + r + \theta))}{S^{\tau_2+1}} \right) \\
 L(Q_2(t)) &= \frac{1}{S^{\tau_5}} \left( \frac{-(n + \eta + \mu)m_2\theta - m_5(n + \eta + \mu)}{S^{\tau_5+1}} + \frac{\theta(Bm_1m_4 - m_2(\mu + \rho + r + \theta))}{S^{\tau_2+1}} \right) \\
 L(H_2(t)) &= \frac{1}{S^{\tau_6}} \left( \frac{q(1 + y)(m_5n - m_2r + m_4q(1 + y) - m_4(b + \mu))}{S^{\tau_4+1}} \right) + \frac{1}{S^{\tau_6}} \left( \frac{-(k_2 + \mu)(m_4q(1 + y) - m_6(k_2 + \mu))}{S^{\tau_6+1}} \right)
 \end{aligned}
 \tag{33}$$

Applying the inverse Laplace Transform to equation (34) results to:

$$\begin{aligned}
 S_2(t) &= t^{\tau_1} \left[ \frac{t^{\tau_1} \mu(Bm_1m_4 + m_1\mu + \lambda(1 - v))}{\Gamma(2\tau_1+1)} + \frac{(\lambda - \lambda v)\Gamma(1 + \tau_4)\Gamma(2\tau_1 + \tau_4 + 1)}{\Gamma(2\tau_1 + \tau_4 + 1)\Gamma(1 + \tau_4)\Gamma(1 + \tau_1)} \right] + \left[ \frac{-Bt^{\tau_1 + \tau_4}(bm_4 - m_5n + m_4q - m_2r + m_4qy + m_4\delta_2 + m_4\mu)(Bm_1m_4 + m_1\mu + \lambda(-1 + v))\Gamma(2\tau_1 + \tau_4 + 1)}{\Gamma(2\tau_1 + \tau_4 + 1)\Gamma(1 + \tau_4)\Gamma(1 + \tau_1)} \right] \\
 E_2(t) &= \left[ \frac{t^{2\tau_2}(r + \theta + \mu + \rho)(-Bm_1m_4 - m_2(r + \theta + \mu + \rho))}{\Gamma(2\tau_2+1)} \right] + \left[ \frac{Bt^{\tau_1 + \tau_4}(bm_4 - m_5n + m_4q - m_2r + m_4qy + m_4\delta_2 + m_4\mu)(Bm_1m_4 + m_1\mu + \lambda(-1 + v))\Gamma(\tau_1 + \tau_4 + 1)}{\Gamma(\tau_1 + \tau_4 + \tau_2 + 1)\Gamma(1 + \tau_4)\Gamma(1 + \tau_1)} \right] \\
 A_2(t) &= t^{\tau_2 + \tau_3} \frac{\rho(-Bm_1m_4 + m_2(r + \theta + \mu + \rho))}{\Gamma(\tau_2 + \tau_3 + 1)} + \frac{t^{2\tau_3}(k_1 + \mu)(k_1m_3 + m_3\mu - m_2\rho)}{\Gamma(2\tau_3 + 1)} \\
 P_2(t) &= t^{\tau_4} \left( \frac{Bm_1m_2r t^{\tau_2}}{\Gamma(\tau_2 + \tau_4 + 1)} - \frac{m_2r t^{\tau_2}(r + \theta + \mu + \rho)}{\Gamma(\tau_2 + \tau_4 + 1)} \right) + t^{\tau_4} \left( \frac{t^{\tau_4}(b + q + y + \mu)(bm_4 - m_5n + m_4q - m_2r + m_4qy + m_4\delta_2 + m_4\mu)}{\Gamma(2\tau_4 + 1)} \right) - t^{\tau_4} \left( \frac{nt^{\tau_5}(-m_2\theta + m_5(n + \eta + \mu))}{\Gamma(\tau_5 + \tau_4 + 1)} \right) \\
 Q_2(t) &= \frac{-t^{\tau_2 + \tau_5}\theta(-Bm_1m_4 - m_2(r + \theta + \mu + \rho))}{\Gamma(\tau_5 + \tau_2 + 1)} + \frac{t^{2\tau_5}(-m_2\theta + m_5(n + \eta + \mu))}{\Gamma(2\tau_5 + 1)} \\
 H_2(t) &= \frac{-qt^{\tau_4 + \tau_6}(1 + y)(bm_4 - m_5n + m_4q - m_2r + m_4qy + m_4\delta_2 + m_4\mu)}{\Gamma(\tau_4 + \tau_6 + 1)} + \frac{t^{2\tau_6}(k_2 + \mu)(k_2m_6 - m_4q(1 + y) - m_6\mu)}{\Gamma(2\tau_6 + 1)}
 \end{aligned}
 \tag{34}$$

### 8. RESULTS AND DISCUSSION



Figures 1 to 6 present the results of numerical investigations conducted on the dynamics of a proposed fractional order transmission model for Covid-19. These figures visually depict the behavior of different populations (Susceptible, Exposed, Asymptomatic, Infected, Quarantine, and Hospitalized) over time (t). The study compared various fractional orders ( $\tau$ ) with the traditional integer order ( $\tau = 1$ ).

The main finding of the study revealed that by varying the fractional order ( $\tau$ ) between 0 and 1, the fractional order Covid-19 model demonstrated increased flexibility in capturing the intricate dynamics of the system. This enhanced flexibility was observed across all populations, as they exhibited an upward trend over time (t), indicating a larger number of individuals transitioning to their respective classes (Susceptible, Exposed, Asymptomatic, Infected, Quarantine, or Hospitalized) at a rate influenced by the recovery process.

Further analysis of the results showed that Figure 2 displayed a gradual reduction in the number of exposed individuals over time. Figure 3 demonstrated a slower spread of the virus within the Asymptomatic population, with a gradual decline in the infection rate. Similarly, Figure 4 indicated a slow decrease in the infection rate among the infected population, as more individuals recovered and moved out of the infected class, contributing to overall containment. The Quarantine population, as shown in Figure 5, initially experienced a decrease, followed by a slower decline over time (t), suggesting a gradual release of individuals from quarantine due to increased control measures. Likewise, the Hospitalized population exhibited a gradual decrease in the infection rate as individuals moved into the Hospitalized class through recovery.

In summary, our research highlights the importance of incorporating fractional order derivatives to assess the impact of control measures in eradicating Covid-19 using a novel model. This inclusion provides greater flexibility and a more accurate representation of the complex dynamics observed in real-world scenarios. Our findings consistently support the notion that

fractional orders contribute to an improved understanding and modeling of the Covid-19 pandemic, enhancing our ability to predict and manage its impact more effectively.

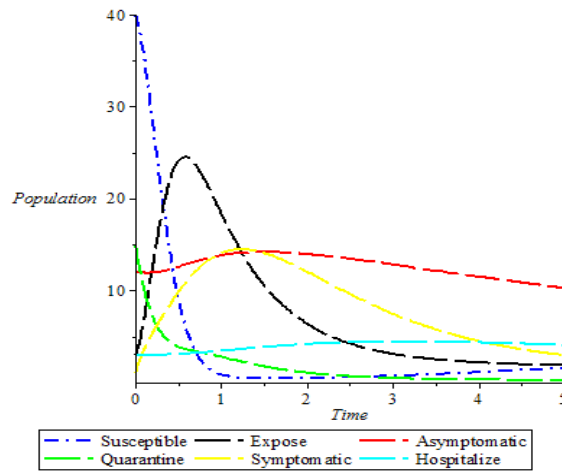


Figure 7 Graph of the population of the model when vaccination is implemented at a rate of 0.6.

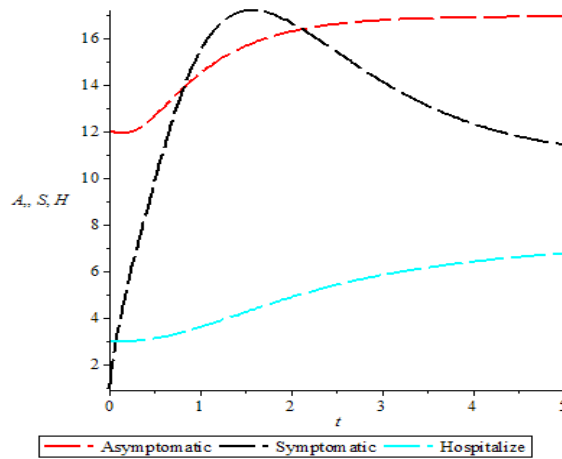


Figure 8 Graph of Asymptomatic, Symptomatic and Hospitalize population Figure Graph of Asymptomatic, when  $y=0.9$

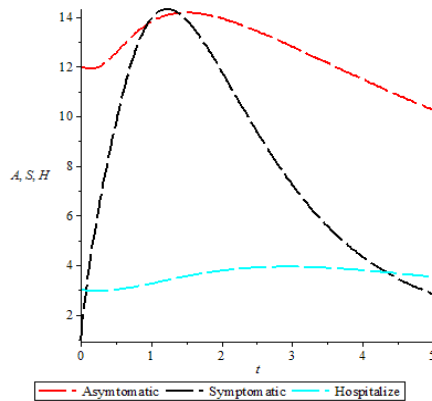


Figure 9 Symptomatic and Hospitalize population when  $y=0$ .

## 9. Conclusions

In conclusion, our study underscores the importance of a well-executed enlightenment program in controlling the transmission of COVID-19. The findings of the model support the notion that an effective awareness campaign, coupled with appropriate implementation measures, can significantly contribute to mitigating the impact of the disease.

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